

PHOTOGRAMMETRY

1.0 Introduction

- Photogrammetry or aerial surveying is a method of surveying generally used to cover large area.
- Any type of imagery – photographs, digital or CCD images, images using thermal or other scanning systems from a satellite etc. may be used in this method.
- The term remote sensing refers to accessing information about an object without being in physical contact with it.
- Thus obtaining the photographs or images through other scanning systems can be classified as remote sensing.
- The purpose of remote sensing may be to acquire qualitative as well as quantitative information about the object land.
- The technique which is used to derive information from the photographs is termed as photo-interpretation and is covered under aerial surveying.
- The photogrammetry which started as terrestrial photogrammetry has now become popular as aerial photogrammetry.

1.1 Terrestrial Photogrammetry

1.1.1 Phototheodolite

- Terrestrial photography is done with a phototheodolite.
- A phototheodolite consists of a camera mounted on a levelling base, such as a theodolite with a telescope on the top of the camera box.
- The line of collimation of the telescope and the optical axis of the camera must lie in the same vertical plane.
- The phototheodolite also has a horizontal plate for measuring horizontal angles.
- The camera has a frame with cross hairs inside.
- The line joining the intersection of cross hairs and the optical centre of the camera lens defines the line of collimation.

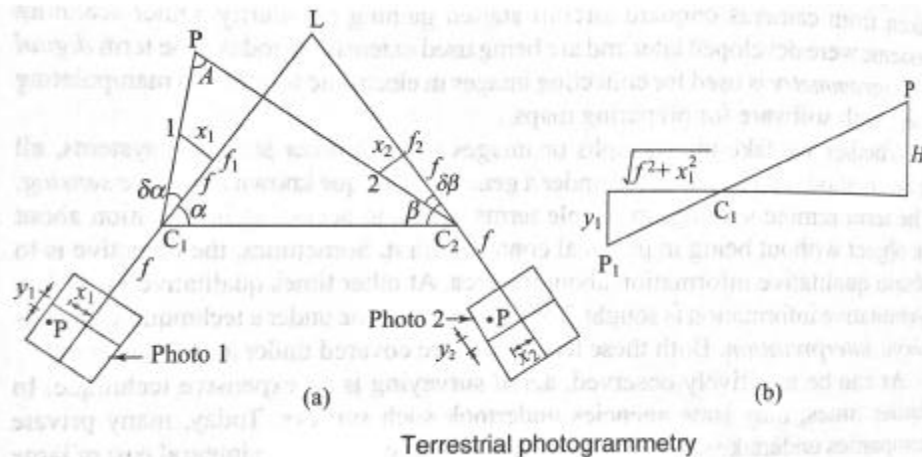


1.1.2 Fieldwork

- Fieldwork in terrestrial photogrammetry includes reconnaissance, establishing control points and taking photographs.
- Reconnaissance is done to locate triangulation stations and stations for placing the cameras.
- The area is thoroughly surveyed so that a minimum number of photographs is required to cover it.
- Triangulation stations can also be used as camera stations.
- The camera stations are so selected that the features to be located are clearly visible in at least two photographs.
- The base line joining the camera stations, which is carefully measured, and the lines of collimation from the cameras must make well formed triangles.
- The photographs should also contain a triangulation station or any other station whose location is already available on a map.

➤ Basic principle of terrestrial photography can be explained by the following example situation.

- Fig. below shows two camera stations C_1 and C_2 . C_1L and C_2L are the line of collimation of the cameras at the two stations intersecting at L .
- Let P be the point to be located in the field on the photos taken as shown.
- C_1C_2 is the base line that has been measured accurately.
- Let α and β be the angles made by the lines of collimation with the base line.
- The coordinates of point P measured on the photographs are (x_1, y_1) from the camera at C_1 and (x_2, y_2) from camera at C_2 .
- Let f be the focal length of the camera.



- To locate the point P on the plan map, the following procedure is followed.
 - (i) On the given plan, locate the camera stations C_1 and C_2 .
 - (ii) Plot angles α and β to locate L and line of collimation of the cameras.

- (iii) Mark points f_1 and f_2 by measuring the focal length of the camera in the direction of line of collimation.
- (iv) From the photograph, measure the coordinate distances. Mark the distances x_1 and x_2 at right angles to lines of collimation from f_1 and f_2 .
- (v) Extend C_1 and C_2 to intersect at P. P is the location of the point in the plan.

(vi) Analytical Procedure;

Let $\angle PC_1L$ be $\delta\alpha$ and $\angle PC_2L$ be $\delta\beta$. It is clear from the figure that $\tan \delta\alpha = x_1/f$ and $\tan \delta\beta = x_2/f$.

$\angle PC_1C_2 = \alpha + \delta\alpha$ and $\angle C_1PC_2 = A = 180^\circ - (\alpha + \delta\alpha) - (\beta - \delta\beta)$ respectively.

Knowing the length C_1C_2 , the lengths C_1P and C_2P can be calculated using the sine rule:

$$C_1P = C_1C_2 \sin(\beta - \delta\beta) / \sin A \text{ and } C_2P = C_1C_2 \sin(\alpha + \delta\alpha) / \sin A$$

(vii) To determine the elevation of point P above the line of collimation at C_1 .

Fig. (b) shows the elevation above the line of collimation.

(viii) From similarity of triangles

$$H = \frac{C_1P}{\sqrt{f^2 + x_1^2}} y_1$$

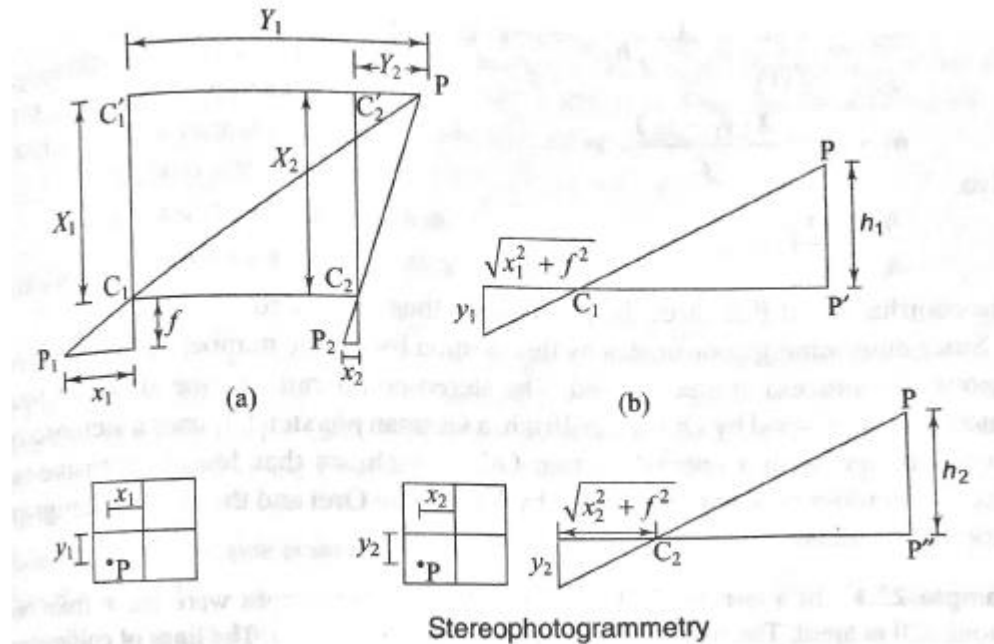
(ix) C_1P can be measured from the plan to scale. RL of P can be found if RL of line of collimation of camera C_1 is known.

1.2 Terrestrial Stereo-photogrammetry

- An advanced method of terrestrial photogrammetry is to take photographs in pairs from the ends of base lines.
- Photographs are generally taken with the optical axis at right angles to the base line.
- This is achieved by taking a backsight to the first station from the second station.
- The camera stations are selected in such a way that the required feature appears in both the photographs.
- The camera stations are located by triangulations.
- The base line joining the camera stations should be measured accurately.
- When the two photographs are properly adjusted for viewing and seen through a stereoscope, the ground features are seen in three dimensions.

1.2.1 Principle of plotting

- Plotting is generally done mechanically using plotting machines.
- The principle is explained here with the help of following fig.



- Let the coordinates of P with respect to C_1 and C_2 be (X_1, Y_1) and (X_2, Y_2) respectively.
- Also, it is clear that $X_1 = X_2$; $C_1'P = Y_1$ and $C_2'P = Y_2$. Length of the Camera base line = $C_1C_2 = Y_1 - Y_2$. Let (x_1, y_1) be the coordinates of P on photograph taken from C_1 and (x_2, y_2) on the photograph taken from C_2 .

➤ At station C_1 , from the similar triangles,

$$\frac{Y_1}{X_1} = \frac{x_1}{f}$$

$$Y_1 = \frac{X_1 x_1}{f}$$

Similarly, at station C_2 ,

$$\frac{Y_2}{X_2} = \frac{x_2}{f}$$

$$Y_2 = \frac{X_2 x_2}{f} = \frac{X_1 x_2}{f} \quad (\text{Since } X_1 = X_2)$$

From this, the base line length

$$C_1 C_2 = Y_1 - Y_2 = \frac{X_1(x_1 - x_2)}{f}$$

Let base line length $C_1 C_2 = B$

Then, $X_1 = \frac{Bf}{x_1 - x_2}$

Also, $Y_1 = \frac{X_1 x_1}{f} = \frac{Bx_1}{x_1 - x_2}$

And $Y_2 = \frac{X_1 x_2}{f} = \frac{Bx_2}{x_1 - x_2}$

To determine the elevation of P, let h_1 be the height of P above the optical axis at C_1 . From the fig (b) considering the similar triangles,

$$\frac{h_1}{y_1} = \frac{C_1 P}{C_1 P_1} = \frac{X_1}{f}, \quad h_1 = \frac{y_1 X_1}{f} = \frac{y_1 X}{f} \quad (X = X_1 = X_2)$$

$$\frac{h_2}{y_2} = \frac{C_2 P}{C_2 P_2} = \frac{X_2}{f}, \quad h_2 = \frac{y_2 X_2}{f} = \frac{y_2 X}{f}$$

$$h_1 - h_2 = \frac{X(y_1 - y_2)}{f}$$

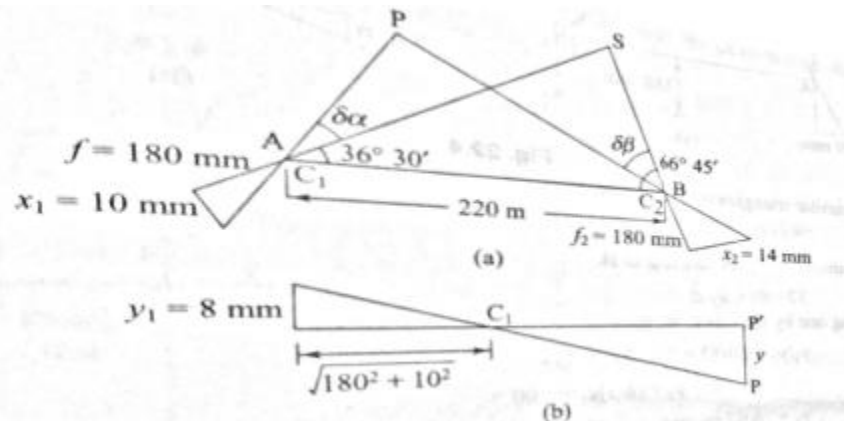
Also, $\frac{h_1}{h_2} = \frac{y_1}{y_2}$

The coordinates of P in three dimensions are thus computed.

- Since determining coordinates by this method for a large number of points is very tedious, the calculation process is mechanized.
- Stereo-comparator, stereo-autograph and stereo-planigraph are a few such machines available commercially for use.

Problems for practice

Q.1. In a terrestrial photographic survey, photographs were taken from two stations 220 m apart. The focal length of the camera was 180 mm. The lines of collimation from the camera stations made angles of $36^\circ 30'$ and $66^\circ 45'$ from the left and right stations, respectively, while observing station S. A point P appeared on the photographs from both stations. P was 10 mm to the left and 8 mm above the cross lines in the first photograph and 14 mm to the left and 3 mm above the cross lines in the second photograph. The reduced level of camera axis at the first station was found to be 182.5 m by observations to the bench mark. Find the coordinates and RL of point P.



- Q. 2. A and B are two camera stations 200 m apart. Stereo-pairs were taken with optical axis at right angles to the camera base line. In the photograph exposed at A, a point P was found to be 20 mm to the right and 8 mm above the cross lines. The same point was 32 mm to the left and 12 mm above the cross lines in the photograph taken from B. If the focal length of the camera lens was 180 mm, find the coordinates of P with respect to origin at A.
- Q. 3. Photographs were taken from two ground stations 200 m apart with a camera having focal length of 150 mm. The camera axis made an angle of $42^{\circ}36'$ at the left station and $60^{\circ}48'$ at the right station with the base line. On the photograph taken from the left station, a point M was 12 mm to the left and 8 mm above the cross lines. On the photograph taken from the right station, M was 16 mm to the left and 6 mm above the cross hairs. The reduced level of the camera axis at the left station was 106.85 m. Find the coordinates of M with reference to the left station and its reduced level.
- Q. 4. Photographs were taken from camera stations 300 m apart, the camera axis being at right angle to the base line at both stations. The focal length of camera was 200 mm. On the photograph taken from the left station, a point appears 32.5 mm to the right and 18.5 mm above the cross lines. On the photograph taken from the right station, the same point appears 68.5 mm to the right and 12 mm above the cross lines. Find the coordinates of the point with the left station as origin.

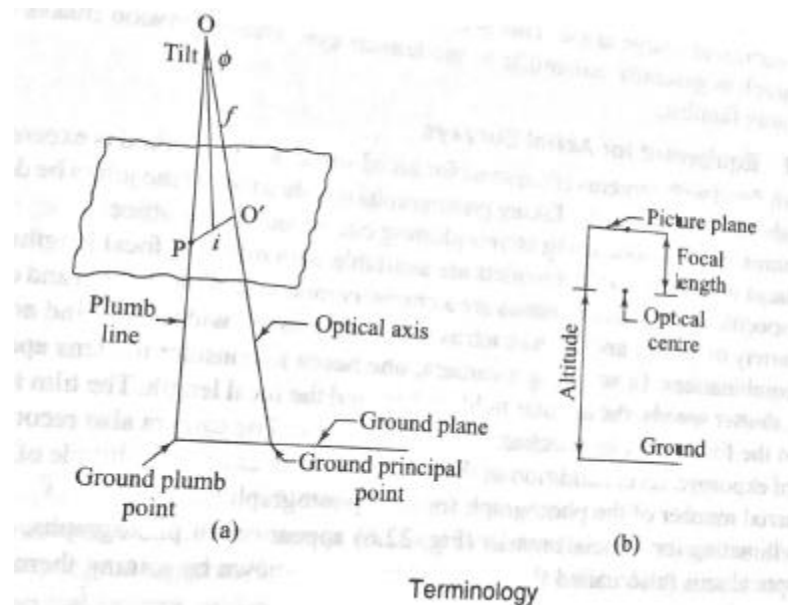
1.3 Aerial Photogrammetry

- Terrestrial photogrammetry is almost out of use with the introduction of aerial techniques.
- Aerial photogrammetry makes use of cameras fitted in an aircraft to photograph an area from an overhead position.
- The principle of stereoscopic vision is used in studying and interpreting aerial photographs.
- Therefore overlapping photographs are taken in the direction of flight as well as in the lateral direction as aircraft flies along a parallel path.
- It is also to be understood that while a map is orthographic projection by projecting points perpendicular to the plane, a photograph is a perspective projection, as all the light rays for forming the image pass through a point.

1.3.1 Basic Terminology

- **Altitude:** Height of the aircraft above the ground.
- **Flying height:** Height of the aircraft above a chosen datum.

- **Exposure station:** Position of the aircraft at the time of exposure of the film. It is essentially the position of the optical centre of the camera lens when film is exposed.
- **Air base:** Distance between two consecutive exposure stations.
- **Tilt and tip:** Tilt is an angle produced by the rotation of aerial camera about line of flight (or vertical axis). It is also the angle which the plane of tilted photograph makes with the plane of vertical photograph. **Tip** is angle produced by the rotation of aerial camera about horizontal axis which is perpendicular to the line of flight. Tip is also called as swing.



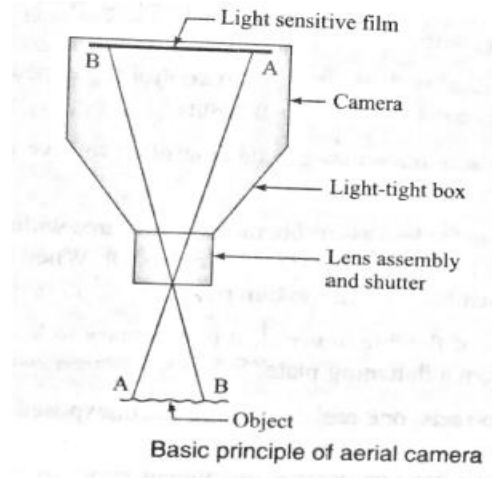
- **Picture plane:** The plane that contains the image at the time of camera exposure.
- **Ground plane:** Horizontal surface from which heights can be measured and which can be used as datum surface.
- **Perspective centre:** Optical centre of the camera through which all rays of light pass.
- **Principal point:** Point of intersection of the optical axis of the camera with the photographic plane. O is the optical centre and O' is the principal point. When the optical axis is extended downwards, the point of intersection with the ground surface is known as principal ground point.
- **Isocentre:** Point on the photograph at which the bisector of the angle of tilt meets the photographic plane. In the fig shown on the previous slide 'i' is the isocentre, at a distance of $f \cos \phi$ along the principal line, where f is the focal length of the camera.
- **Plumb points:** The points at which the vertical line through the optical centre meets the photographic plane and the ground surface are the plumb points. The plumb point on the ground surface is also known as ground nadir point and that on the photograph is known as nadir point.

- **Homologous points:** Points on the ground and their representations in the photograph in perspective projection.
- **Vertical photograph:** Photograph obtained with an aerial camera when the ground is perfectly flat and the optical axis is vertical. This rarely obtained.
- **Tilted photograph:** Photograph obtained when the optical axis is inclined unintentionally to the vertical at an angle not greater than 3° .
- **Oblique photograph:** Photograph taken with the optical axis inclined to the vertical at a large angle. This is done because the view as seen in a vertical photograph is generally unfamiliar to the human eye. The inclination makes the view more familiar.

1.3.2 Equipment for Aerial Surveys:

- Aircraft fitted cameras is required for aerial surveys. The method is expensive but viable for large projects.
- Taking photographs is only one part of the job to be done. A lot more work is to be done for stereo plotting in the office with proper equipment.

- **Aerial Camera:** There is no basic difference in working principle between an ordinary camera and aerial camera. There are several types of cameras in use in aerial photography including digital cameras. The basic principle of working is explained in the fig. below.



- **Aperture:** It is opening of the shutter at the time of exposure, which can be adjusted, also called f-stop. The f-stop numbers are given as $f/8$, $f/11$ etc. which is focal length divided by effective diameter of the lens opening.
- **Shutter speed:** It is the time the shutter remains open to allow the light to fall on the film. This can vary from $1/2000$ to 1 or more seconds.
- **Film speed:** It is indicated by a number as 100, 200 etc. and means the amount of radiation required for the film response. Larger number indicates large speed, less aperture and small shutter speeds for a given response.
- **Focus:** It is the operation for bringing the image clear and is done by varying the distance between the lens and the image plane. When the camera is focused, a clear image of the object will be formed on the plane of the film.

1.3.3 Procedure for Aerial Surveying

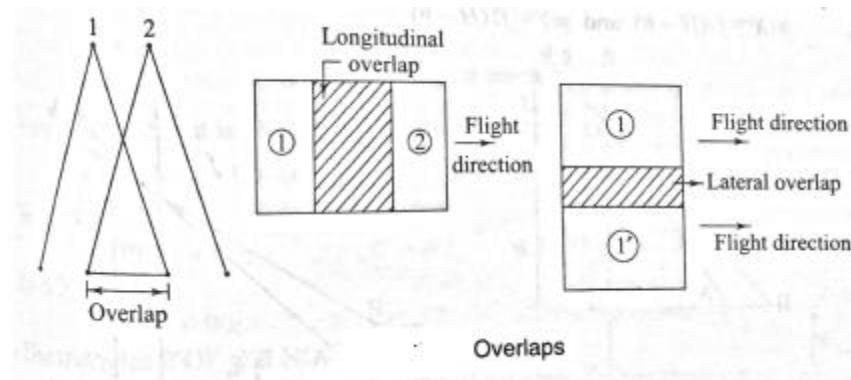
- The procedure for aerial surveying includes reconnaissance of the area, establishing ground controls, flight planning, photography and paperwork including computation and plotting.
- **Reconnaissance** is undertaken to study the important features of the ground for reference purposes.
- **Ground control points** are required in order to obtain a set of points of known positions based on which other points are located or plotted.
- The number of ground control points depends upon the extent of area covered, scale of the map, flight plan and the process of preparing the map.
- A minimum three control points must appear in each photograph.
- In the earlier days of aerial photography, triangulation or precise traverse survey was required to establish control points.
- These points should be seen prominent and identifiable in the photographs.
- Both horizontal and vertical control points must be established. Thus the ground coordinates and elevations of these points are known.
- In modern methods of aerial photography, a number of new technologies are available with total stations and satellite GPS navigators.
- The ground control points can be established to great accuracy using these new devices.
- Coordinates in terms of latitude, longitude and elevations can easily be established by these instruments.

- **Flight planning** includes the decision on the speed of the aircraft, altitude, area covered by each exposure and the overlaps in the direction of flight and in the traverse direction, number of flights, number of exposures and scale of photographs.
- For the above, the flight plan considers the type of camera and its focal length, scale of the photograph, altitude, speed of aircraft, overlaps of the photographs, area covered by each photograph, time interval between exposures and the number of photographs required.

- **Overlaps** are required to ensure complete coverage of the area.
- Stereoscopic vision is possible only with overlapped parts of the photographs (parts common to two photographs).
- Overlaps also ensure that the central part of the photograph is less distorted than the outer edges.
- The photographs are made to overlap longitudinally as well as laterally.
- Lateral overlap ensures that no area is left without being photographed.
- Longitudinal overlap is generally 60% while lateral overlap is about 30%.

- To obtain overlaps, the aircraft flies in a straight line to the extent possible and camera provides exposures as per desired time intervals.
- The interval between the exposures must be such that the desired longitudinal overlap of the area covered is attained.
- The next flight is undertaken in parallel line such that the required lateral overlap is obtained.

- Fig below shows how longitudinal and lateral overlaps are obtained.



Scale of photographs

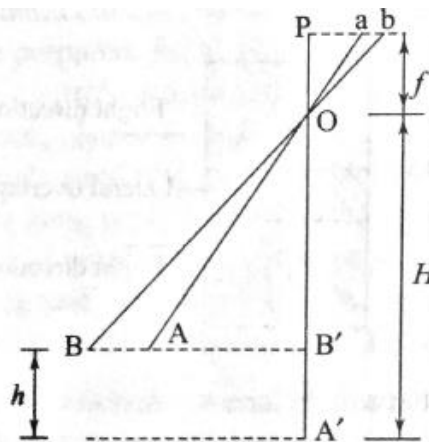
- The representation of any object in the photograph depends upon the distance of the object from the camera.
- In aerial surveying this distance is represented by flying height.
- As all the ground features do not have the same elevation, it is obvious the scale representation will be different for features lying at different elevations.
- A photograph unlike a map (based on orthographic projection) does not have uniform scale.
- The scale can be varied by changing the flying height.
- To account for changes in scale due to ground relief (differing elevation), an average elevation and average scale can be worked out.

Vertical photographs:

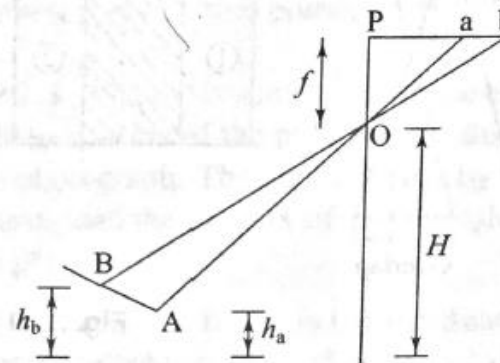
- If the ground is perfectly plane, the photograph obtained will be similar to a map having uniform scale.
- Referring to the fig. below, the length AB (on plane ground) on the ground is represented by ab in the photograph. So, the scale S of the photograph is ab/AB and from similar triangles abO and ABO ;

$$S = ab/AB = f/(H-h)$$

where f is the focal length of the camera, H is the flying height and h is the elevation of line AB. H and h are measured from the same datum. If A and B are at different levels [Fig (b)] with elevations h_a and h_b an average elevation for the datum can be used; $h_{av} = (h_a + h_b)/2$. Scale for line AB in the photograph can be taken as $f / (H - h_{av})$. In case of an area, the elevations of large number of points can be averaged [$h_{av} = (h_1+h_2+h_3+\dots+h_n)/n$]



(a) A and B at the same level



(b) A and B at different levels

Scale of photograph

- If the flying height also changes during the coverage of an area, an average flying height can also be worked out and used in the formula for the scale.
- A vertical photograph, thus, does not have a uniform real scale owing to ground relief.
- The length of line can be determined using the coordinates on the photograph and on the ground. The ground coordinates are determined with reference to the ground plumb point. The photograph coordinates are determined from the principal point of the photograph.
- If (X, Y) are the ground coordinates and (x, y) are the photographic coordinates then, $(x/ X) = (y/ Y) = f/ (H-h)$.
- If PQ is a line, (X_p, Y_p) are the ground coordinates of P and (x_p, y_p) are the photographic coordinates of P and (X_q, Y_q) and (x_q, y_q) represents the similar terms for Q.

$$X_p = \frac{(H - h_p)x_p}{f}$$

$$Y_p = \frac{(H - h_p)y_p}{f}$$

$$X_q = \frac{(H - h_q)x_q}{f}$$

$$Y_q = \frac{(H - h_q)y_q}{f}$$

$$\text{Length } PQ = \sqrt{(X_p - X_q)^2 + (Y_p - Y_q)^2}$$

Tilted photographs

- In case of tilted photographs, the scale factor is more difficult to derive.
- If ground relief is also to be taken into account, the determination of scale becomes very complex.

Fig. shows a tilted photograph with angle of tilt 't'.

Let k = the principal point; n = nadir point; and

a = the image point of ground point A

nk is the principal line. Draw perpendicular am to the Principal line and mm' perpendicular to the plumb line.

Since the lines am and mm' are horizontal, the plane amm' is a horizontal plane.

Since $\Delta Om'a$ is similar to ΔONA , then

$$\frac{m'a}{NA} = \frac{Om'}{ON} = \frac{\text{Map distance}}{\text{Ground distance}} = S \text{ (scale)}$$

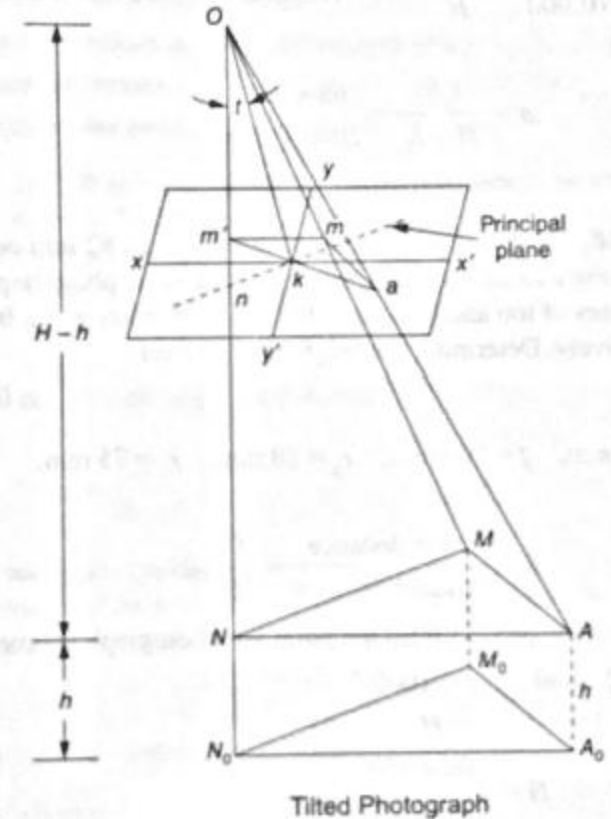
But, $Om' = On - m'n = f \sec t - mn \sin t$

where, f is the focal length Ok .

If the ground point A is at height h above datum,

$$ON = ON_0 - NN_0 = H - h$$

$$S = \frac{f \sec t - mn \sin t}{H - h}$$



Let the photographic coordinates of the image a be x and y as shown in the fig.

Now, mn is on the principal axes y' which makes clockwise angle θ with y -axis.

$$x' = mb + ab = kd + ab$$

$$= x \cos \theta + y \sin \theta$$

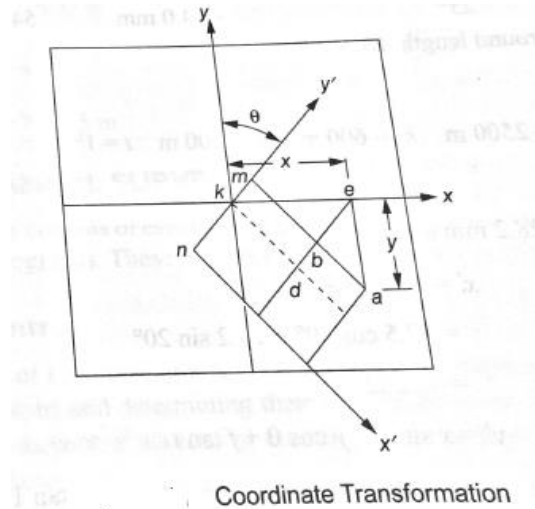
and

$$y' = km + kn$$

$$= ed - eb + kn$$

$$= x \sin \theta - y \cos \theta + f \tan \theta$$

$$S = \frac{f \sec t - y' \sin t}{H - h}$$



- From the above equation, it may be observed that the scale is same for all the points along the line ma since the value of y' is the same.
- To find the scale at a given point on tilted photograph, the following data is required: (i) focal length f (ii) tilt t (iii) height of flight H (iv) height of point h (v) swing angle θ .

Ground Coordinates and Length

- The coordinates x' and y' may be measured on the photograph. The calculation is done as:

$$x' = am$$

$$\begin{aligned} X = AM &= \frac{x'}{\text{scale}} \\ &= \frac{H - h}{f \sec t - y' \sin t} x' \end{aligned}$$

$$m'm = mn \cos t = y' \cos t$$

$$Y = NM = \frac{y' \cos t}{\text{scale}}$$

$$= \frac{H - h}{f \sec t - y' \sin t} y' \cos t$$

Thus, from photograph coordinates, the ground coordinates can be found. If the photograph coordinates of two points A and B are known, the ground distance between A and B is given by,

$$L = \sqrt{(X_a - X_b)^2 + (Y_a - Y_b)^2}$$

Practice Problem

The reading on a photograph with 1° tilt on point A and B having elevations of 600 m and 200 m above datum with a camera of focal length 220 mm and flying altitude of 2500 m above the datum are shown below.

Point	Angle between y-axis and principal line	Photographic coordinates	
		x (mm)	y (mm)
A	20°	42.5	28.2
B	25°	-24.0	54.0

Determine the ground length AB.

Number of photographs to cover a given area

Let

P_l = Longitudinal overlap

P_w = Side overlap

L = Ground Length

W = Width of ground

l = Length of photograph in flight direction

w = Width of photograph

S = Scale (photo to ground)

Then,

$$L = (1 - P_l)Sl$$

$$W = (1 - P_w)Sw$$

Ground area covered by each photograph

$$= LW = (1 - P_l)(1 - P_w)S^2lw$$

Number of photographs required to cover the area

$$= \frac{\text{Area of ground}}{\text{Area covered by a photograph}}$$

Number of photographs required to cover a given area of size L×B

Net length covered by each photograph = $(1 - P_l)Sl$

∴ Number of photographs required along a strip

$$N_1 = \frac{L}{(1 - P_l)Sl} + 1$$

Similarly net width covered by each photograph = $(1 - P_w)Sw$

$$\therefore N_2 = \frac{B}{(1 - P_w)Sw} + 1$$

∴ Total number of photographs required = N_1N_2

Interval between exposure

Let v km/h be the uniform velocity of the aircraft

Air base be L' km

Then time required to cover air base L' , i.e., the distance between two exposure is

$$T = \frac{L'}{v} \text{ hours} = 3600 \frac{L'}{v} \text{ seconds}$$

Altitude of Aircraft

- The selection of height above ground depends upon the desired contour intervals of the finished map.
- Defining C-factor as the number by which the contour interval is multiplied to obtain the maximum height above the ground,
$$\text{Flying height} = \text{Contour Interval} \times \text{C-factor}$$
- C-factor vary from 500 to 1500 depending upon the conditions surrounding the map compilation operation.

Practice Problems

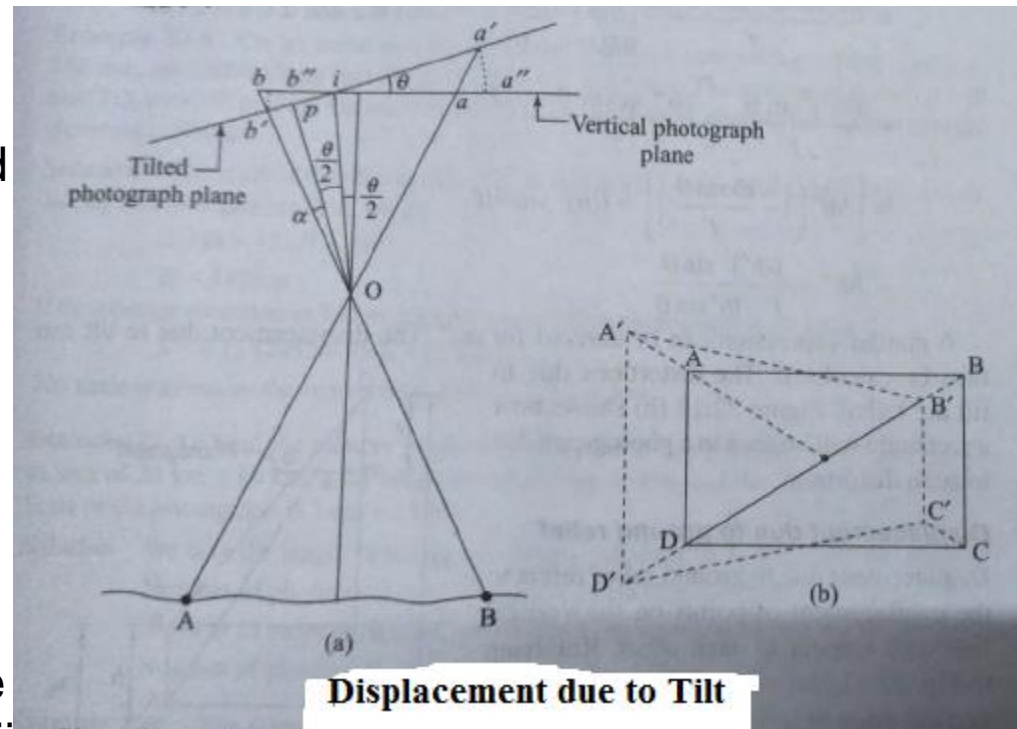
1. The scale of a photograph is 1: 10000 and the size of the photograph is $200 \times 200 \text{ mm}^2$. Determine the number of photographs required to cover an area of $12 \text{ km} \times 16 \text{ km}$, if the longitudinal lap is 60% and the side lap is 30%.
2. Determine the scale of the photograph and the length of the air base for the data given below:
Focal length of camera = 200mm; Flying height = 2000 m above datum;
Height of the ground above datum = 500 m; Overlap = 60%;
Print size = $250 \times 250 \text{ mm}^2$.
3. An area of $100 \times 100 \text{ km}^2$ is to be surveyed by aerial photographs. The following data is available: Focal length of camera= 200 mm; least count of interval meter=0.5 s; size of photograph = $200 \text{ mm} \times 200 \text{ mm}$; average scale of photograph=1:15000; average elevation of terrain = 400m; longitudinal overlap = 60%; side overlap = 30%; velocity of aircraft = 300 km/h. Determine (i) Flying height; (ii) Spacing of flight lines; (iii) Ground distance between exposure; (iv) Exposure interval; (v) Number of photographs required.

1.4 Distortions in Aerial Photographs

- The photographic positions of objects as taken from cameras differ from their orthographic positions, due to the tilt of the camera axis as well as ground relief.
- These displacements are corrected to some extent before maps are prepared from photographs.

Displacement due to tilt

Fig. beside shows the displacement Due to tilt, where O is the optical centre, points A and B on the ground are seen as a' and b' in the tilted photograph; a and b are the points which would have been obtained in a vertical photograph. The isocentre 'i' is obtained by bisecting the angle of tilt passing through O. p is the principal point. pO is a line perpendicular to the picture plane. The angle of tilt is θ . With i as centre Draw two arcs with ia' and ib' as radii. These intersect the vertical photograph plane at a'' and b'' .



Since $ib' = ib''$ by construction, $\Delta ib'b''$ is isosceles.

In $\angle ib'b''$, angle at $i = \theta$, the tilt angle. The remaining two angles at b' and b'' will be equal, i.e., $90 - \theta/2$. Thus, $\angle ib'b'' = \angle ib''b' = 90 - \theta/2$

Also, $\angle piO = 90 - \theta/2$ (As angle at p is 90°)

If the lines $b'b''$ and Oi are cut by the line $b'a'$, then $\angle piO$ and $\angle ib'b''$ are alternate and equal angles, hence the lines $b'b''$ and Oi are parallel. Thus ΔiOb and $\Delta b''b'b$ are similar triangles. Therefore,

$$\frac{bb''}{b'b''} = \frac{ib}{Oi} \Rightarrow Oi = \frac{f}{\cos(\theta/2)} = f \sec\left(\frac{\theta}{2}\right)$$

From $\Delta ib'b''$,

$$\angle b'ib'' = \theta; \quad \angle ib'b'' = \angle ib''b' = 90 - \theta/2$$

Applying sine rule,

$$\frac{ib'}{\sin\left(90 - \frac{\theta}{2}\right)} = \frac{b'b''}{\sin \theta}$$

$$b'b'' = \frac{ib' \sin \theta}{\sin\left(90 - \frac{\theta}{2}\right)} = 2ib' \sin(\theta/2)$$

$$[\sin(90 - \theta/2) = \cos \theta/2 \text{ and } \sin \theta = 2 \sin\left(\frac{\theta}{2}\right) \cos\left(\frac{\theta}{2}\right)]$$

From earlier equation for similar triangles

$$bb'' = b'b'' \frac{ib}{Oi} = 2ib' \sin(\theta/2) \frac{ib'' + b''b}{f \sec \theta/2}$$

$$bb'' = \frac{ib' \sin \theta (ib'' + b''b)}{f} \quad [\sec \theta/2 = 1/\cos \theta/2 \text{ and } 2 \sin \theta/2 \cos \theta/2 = \sin \theta]$$

$$bb'' \left(1 - \frac{ib' \sin \theta}{f}\right) = \frac{(ib'')^2 \sin \theta}{f}$$

$$bb'' = \frac{(ib'')^2 \sin \theta}{f - ib' \sin \theta}$$

- A similar expression can be derived for aa". The displacement due to tilt can thus be calculated.
- The distortion due to tilt are radial. The fig shows how a rectangle will be seen in a photograph due to such distortion.

Displacement due to ground relief

- Displacement due to ground relief refers to the displacement of points on the vertical line with respect to each other.
- Fig shows that the vertical line AB on the ground is seen in the photograph as ab, displaced radially instead of being superimposed as in map.
- The radial shift may be worked out as below.

H is the flying height,

f is the focal length,

h is the height of B over A. From triangles PbO and OBB';

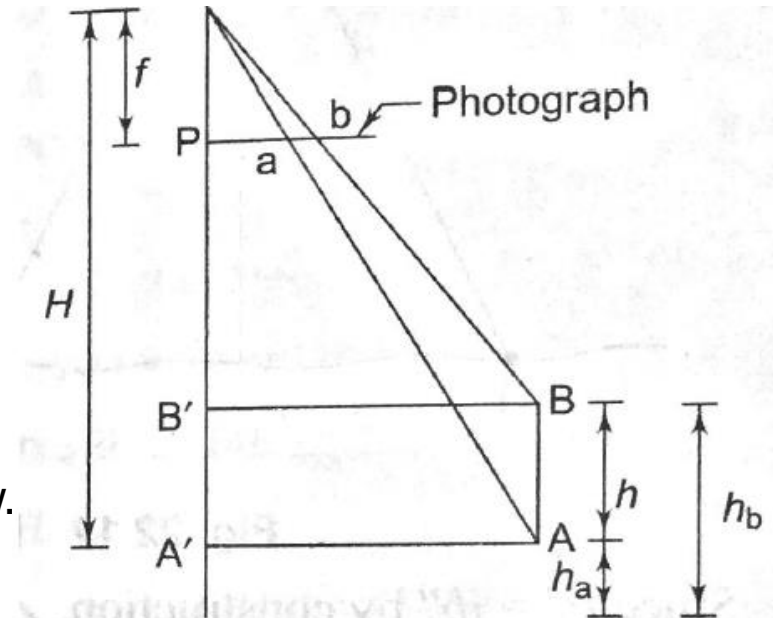
$$\frac{Pb}{f} = \frac{BB'}{H-h} \text{ where } h = h_b - h_a. \text{ Similarly from } \Delta PaO \text{ and } \Delta OAA';$$

$$\frac{Pa}{f} = \frac{BB'}{H}$$

$$\text{Now, } \frac{Pb}{Pa} = \frac{H}{H-h}$$

$$\text{or } \frac{Pb - Pa}{Pa} = \frac{h}{H-h} \text{ [the displacement } ab = Pb - Pa]$$

$$\text{the displacement } ab = h \frac{Pa}{H-h}$$



Displacement due to ground relief

It is assumed that the photograph is vertical.

Problems for practice:

1. Aerial photographs were taken with a camera having a focal length of 180 mm. The average elevation of the ground in the photograph is 160 m. Find (a) the scale of the map if the flying height was 2500 m and (b) the flying height required to have a photo scale of 1 in 6000.
2. On an aerial photograph taken with a camera having a focal length of 150 mm, 1800 m long line PQ had a length of 125 mm. The average elevation of the line PQ was 290 m. Find the scale of another area in the same photograph with an average elevation of 950 m.
3. Find the number of photographs (size 250 mm × 250 mm) required to cover an area of 20 km × 16 km if the longitudinal overlap is 60% and the side overlap is 30%. Scale of the photograph is 1 cm = 150 m.
4. The scale of a photograph of size 250 mm × 250 mm is 1/12000. Determine the number of photographs required to cover an area of 250 km² if the longitudinal and side overlaps have to be 60% and 30% respectively.
5. In a photograph taken with an aerial camera having a focal length of 200 mm, a line PQ measures 108 mm. If the same line has a length of 40 mm on a map drawn to a scale of 1:40000, find the flying height of the aircraft if the average elevation is 400 m.

6. In a vertical photograph, a line 2100 m long measures 110 mm. If the average elevation of the line is 480 m, find the scale of an area in the same photograph with an average elevation of 1200 m. The focal length of the camera used is 210 mm.
7. Points P and Q have elevations of 600 m and 300 m respectively. The photographic coordinates of points P and Q were measured as P(35, 25) and Q(20, 50) in millimeters. The photograph was taken with a camera having a focal length of 210 mm and from an altitude of 2500 m. Find the length of line PQ.
8. The height of a chimney is 110 m. In a vertical photograph taken from an altitude of 2800 m, the chimney appears at a distance of 70 mm from the principal point. Determine the displacement of the top of the chimney from the bottom if the elevation of the base is 1200 m.
9. The image of the base and top of a 200 m tower is seen in a photograph of scale 1:15000. If the base point is 65 mm from the principal point, find coordinate of the top from the principal point. The focal length of the camera was 200 mm and assume that the base of the tower is datum plane.
10. An aerial photo 250 mm × 250 mm was taken with a camera having a focal length of 200 mm. At the time of exposure, the tilt was assumed to be 1.5° . Find the displacement of a point lying at 70 mm on a line passing through the principal point.

1.5 Mosaics

- An assemblage of aerial photographs whose edges have usually been cut and matched to form a continuous photographic representation of the particular area is called a mosaic.
- Generally the mosaic contains the centre portion of the photograph which has generally an average scale.
- If this assemblage is made without any control, then it is called uncontrolled mosaic.
- If, before being laid, the prints have been properly rectified, i.e., enlarged or reduced and fitted on adequate ground control, i.e., to fit pre-determined locations of certain important features, the mosaic is said to be a controlled mosaic.
- The controlled mosaic, though more accurate, retains the changes in scale and displacements due to differences in relief within the individual prints.
- A contoured mosaic shows the relief by means of contours, and may be either controlled or uncontrolled.
- A semi controlled mosaic may be prepared from unrectified photograph assembled to have ground control; alternatively rectified photographs may be used with no ground control.
- Semi-controlled mosaics are a compromise between economy and accuracy.

1.5.1 Photographs and maps:

- An aerial photograph is a two dimensional view of an area at an instant of time. Maps, on the other hand, are made with data collected at some point of time. It takes time to prepare the map and some of the measured data may be outdated by the time the map gets prepared.
- Aerial photographs do not have uniform scale over the whole area. The image is also distorted due to tilt and ground relief. Maps are drawn to a uniform scale.
- A photograph is a perspective projection whereas a map is an orthogonal projection.
- A photograph gives full details of landscape, including the insignificant details. Maps, on the other hand, can be made to exact details the customer wants, i.e., important features required for the project.
- An air photograph shows details as seen by the camera. A map on the other hand, shows details using conventional symbols.

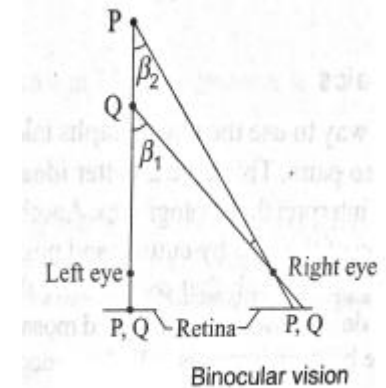
1.6 Stereoscopic Vision

- Stereoscopic vision essentially lends height or depth to the object in the photograph as it would have been seen by the human eye.
- If photographs are available in stereo-pairs, it is possible to get a three dimensional view of the object.

Stereoscopic or binocular vision

- The human brain receives two images of the same object through the two eyes as seen from slightly different angles and fuses them into a 3-d view.

- The base length for vision by human eyes is about 65 mm.
- As shown in the fig. beside PQ is the object. Each eye sees the object but from slightly different angle: β_1 and β_2 known as parallax angles.
- The two images received by the retina are fused into a single image giving depth to the vision.
- When the distance between eyes and object is very large, the parallax angles become smaller and at about 600 m the relative depth is no longer perceived by the eye.



Stereoscopic pairs

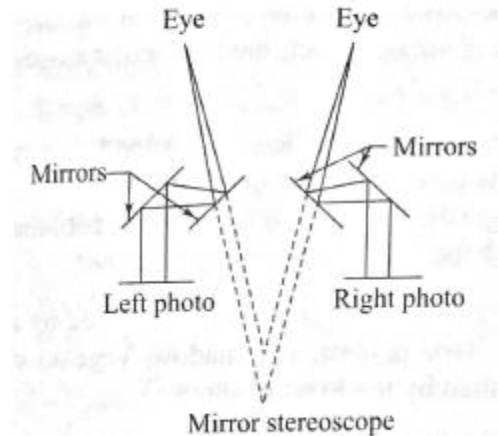
- To enable stereoscopic vision, it is essential to have photographs in pairs covering common areas.
- In aerial photography, photographs are taken from two camera positions with sufficient overlap in the photographs.
- Only the areas seen in both the photographs are amenable to stereoscopic vision.
- The common part of the photographs give us two views of the same object from slightly different angles, as seen by human eyes.
- When the photographs are suitably placed and the left eye views one and the right eye views the other, the common area comes in relief or in three dimensions due to stereoscopic fusion.
- Stereoscopic vision enables one to view the depth or height of points on the photograph.

Stereoscope

- There are many types of stereoscopes: mirror stereoscope, lens stereoscope, scanning mirror and zoom stereoscope. Mirror and lens stereoscopes are handy and commonly used.

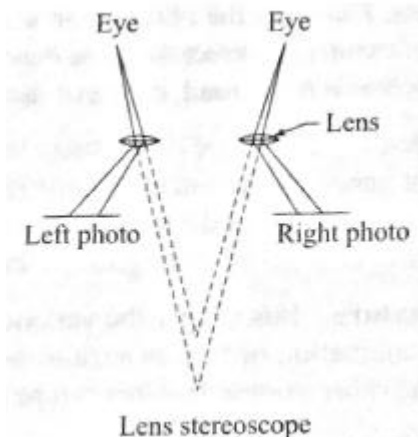
Mirror Stereoscope

- The schematic diagram of mirror stereoscope is shown in the fig. beside.
- It consists of two viewing eyepieces.
- A stereoscopic pair of photographs is placed at a distance from the stereoscope. The photographs are adjusted so that one photograph is seen through one eyepiece.
- As the viewer looks through the stereoscope, he sees the image of the same object (the overlapping part) on the two photographs, the viewer gets a stereoscopic view by fusion.
- The terrain is seen in three dimensional view.



Lens stereoscope

- A lens stereoscope has two eyepieces through which the observer sees the photographs, providing the experience of stereoscopic or spatial view.
- The lenses help to magnify the image as seen by each eye. The distances between the eyepieces is adjustable and can be set by the observer as required.



- This distance is approximately equal to human eyes.
- The lenses tend to magnify the object and its height.
- Lens stereoscope are more compact than mirror stereoscopes.

1.7 Photo-interpretation

- Photo-interpretation refers to the accurate identification of features seen in the photographs.
- It is more difficult to identify objects in vertical photographs than in tilted photographs owing to familiarity of view in oblique photographs.
- Colour photographs are easier to interpret than black and white photographs due to tonal variations.
- A stereoscopic pair is easier to interpret due to depth available in photographs when seen through a stereoscope.
- Interpretation of aerial photographs is required extensively in developmental project design and execution.
- Considerable amount of practice and experience is required to correctly interpret the photographs.

General Features of Photographic Images

Some of the basic features of photographs that help in identification are discussed here.

(i) Size: The size of an object in the photograph is sometimes helpful in interpretation. Knowing the photograph scale, it is possible to have an idea about the size. The size distinguishes the objects of similar shape in photograph such as a river, road, canal or drain.

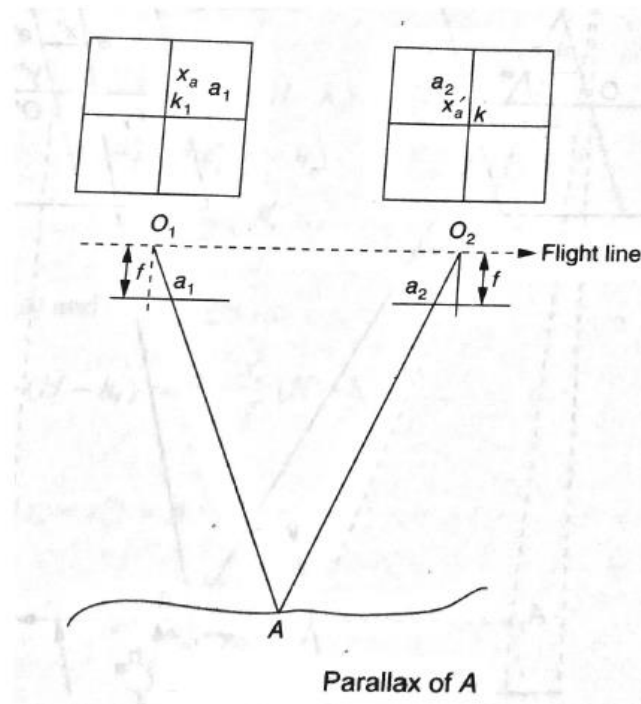
- (ii) Shape:** The shape of an object is helpful in identifying an object. Regular shaped objects are generally man-made. Shape relates to the general outline of the object. A railway line and a roadway can be distinguished from their form. Objects of same size can be distinguished from their shape.
- (iii) Texture:** It is simply variation in tone of the photograph. It is produced by a combination of factors such as size, shape, tone pattern and shadow. Vegetation and other ground features can be distinguished by the tonal changes.
- (iv) Pattern:** It is spatial arrangement of objects in a particular set. A habitat can easily be distinguished by the arrangement of roads, houses etc. because of the pattern.
- (v) Shadow:** The shadow of an object formed during photography is sometimes helpful in identification, as it shows the outline of the object.
- (vi) Tone:** It produced by the amount of light reflected back by the object to the camera. If the particular tone associated with specific objects are known, it is easy to identify them.
- (vii) Location:** The location of an object in the photograph helps in identifying the object itself. Knowing the objects or areas surrounding the object, one can identify the main object.

1.8 Parallax

- Parallax of a point is the displacement of the image of the point on two successive exposures.

a_1 is the position of ground point A in photograph at O_1 .

a_2 is the position of ground point A in photograph at O_2 .



- Taking the line of flight as x-coordinate, let x_a be the coordinate of a_1 in photograph at O_1 and x'_a the coordinate of a_2 in photograph at O_2 . Then

$$\text{Parallax of A } p = x_a - x'_a$$

(Algebraic sign of x_a and x'_a should be taken into consideration, e.g., if $x_a=32.5$ mm and $x'_a=-28.4$ mm, $p = 32.5 - (-28.4) = 60.9$ mm)

1.8.1 Parallax Equations

- Fig beside shows vertical photographs taken from positions O_1 and O_2 at same flying heights H . (x_a, y_a) are the coordinates of image A in photograph taken at O_1 and (x'_a, y'_a) are the coordinates of in photograph taken at O_2 .

$$\text{parallax } p_a = x_a - x'_a$$

Since the flight is in the direction of x-axis,

$$y_a = y'_a$$

- Let X_A and Y_A be the ground coordinates of A which is at an elevation h_a above datum. From similar triangles O_1Oa_y and $O_1A_0A_y$,

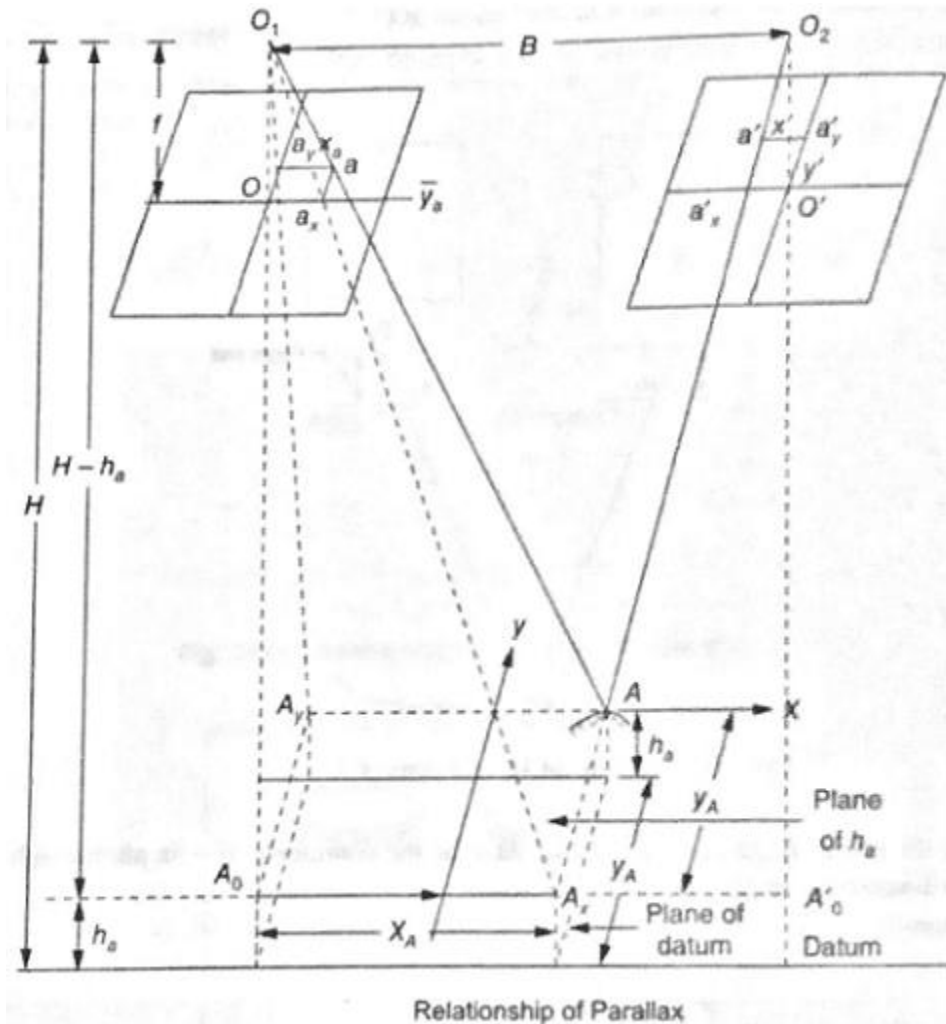
$$\frac{f}{H - h_a} = \frac{y_a}{Y_A} \Rightarrow Y_A = \frac{y_a}{f} (H - h_a)$$

Similarly, from triangles O_1a_xO and $O_1A_0A_x$, we get

$$\frac{f}{H - h_a} = \frac{x_a}{X_A} \Rightarrow X_A = \frac{x_a}{f} (H - h_a) \quad (i)$$

From triangle $O_2O'a'_x$ and $O_2A'_0A_x$

$$\frac{B - X_A}{-x'_a} = \frac{H - h_a}{f} \Rightarrow f(B - X_A) = -x'_a(H - h_a)$$



$$X_a = B + \frac{x'_a}{f} (H - h_a) \quad (ii)$$

From equations (i) and (ii)

$$\frac{x_a}{f} (H - h_a) = B + \frac{x'_a}{f} (H - h_a)$$

$$\text{or } \frac{H - h_a}{f} (x_a - x'_a) = B$$

$$\text{or } H - h_a = \frac{Bf}{x_a - x'_a}$$

$$\text{or } h_a = H - \frac{Bf}{x_a - x'_a} = H - \frac{Bf}{p_a} \quad (iii)$$

Substituting $\frac{Bf}{p_a}$ for $H - h_a$ in Eq. (i) and (ii)

$$X_A = B \frac{x_a}{p_a} \quad (iv) \quad \text{and} \quad Y_A = B \frac{y_a}{p_a} \quad (v)$$

The (iii), (iv) and (v) are known as parallax equations.

1.8.2 Difference in elevation by stereoscopic parallaxes

➤ Consider the parallaxes of the two points A and A₁ which have elevation difference Δh. Let,

$p = \text{parallax of } A$

$p_1 = \text{parallax of } A_1$

The difference of parallaxes

$$\Delta p = p_1 - p$$

By drawing $O_1a' \parallel O_2A$,

a' is located on the photograph

$$p = a'a$$

Also, by drawing $O_1a'_1 \parallel O_2A_1$,

a'_1 is located on the photograph

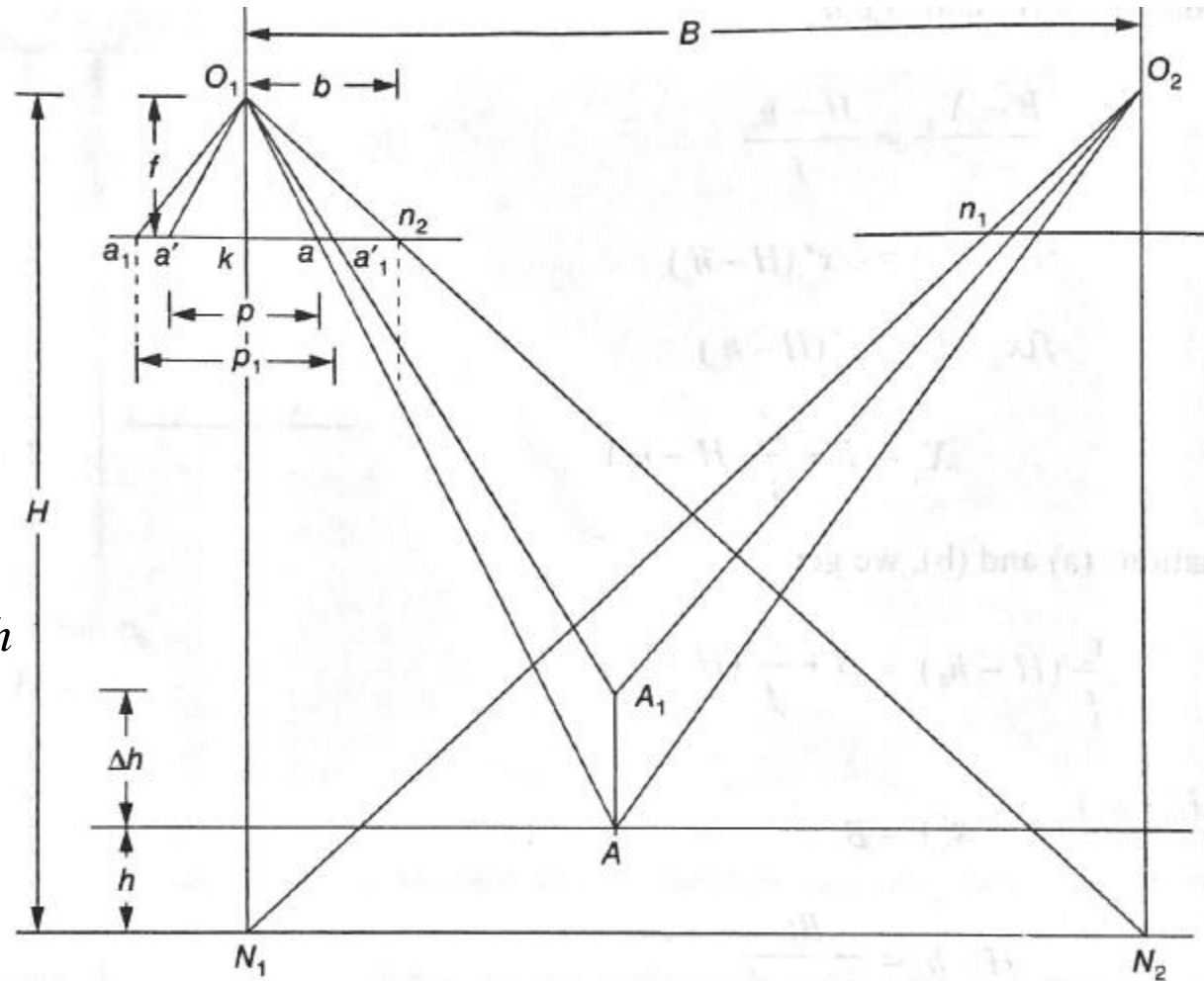
$$p_1 = a'_1a_1$$

From Similar triangles,

$O_1a'a$ and O_1aO_2

$$\frac{f}{H-h} = \frac{p}{B}$$

$$p = \frac{Bf}{H-h} \quad (5)$$



Difference in Elevation by Stereoscopic Parallaxes

From similar triangles O_1kn_2 and $O_1N_1N_2$

$$\frac{f}{H} = \frac{b}{N_1N_2} = \frac{b}{B}$$

$$Bf = bH \quad (6)$$

From Eq (5) and (6)

$$p = \frac{bH}{H - h}$$

hence,

$$p_1 = \frac{bH}{H - h - \Delta h}$$

$$\begin{aligned} \text{Now, } \Delta p &= p_1 - p = bH \left[\frac{1}{H - h - \Delta h} - \frac{1}{H - h} \right] \\ &= bH \frac{\Delta h}{(H - h - \Delta h)(H - h)} \end{aligned}$$

$$\Delta p(H - h - \Delta h)(H - h) = bH \Delta h$$

$$\Delta p(H - h)(H - h) - \Delta p \Delta h(H - h) = bH \Delta h$$

$$\Delta h [bH + \Delta p(H - h)] = \Delta p(H - h)^2$$

$$\Delta h = \frac{\Delta p(H - h)^2}{bH + \Delta p(H - h)}$$

Using the above equation the difference in elevation by stereoscopic parallaxes can be determined.

Problems for Practice

1. Aerial photograph of an object were taken from two positions 3000 m apart. If the object is 400 m above the mean sea level, focal length of the camera is 180 mm and parallax was 270 mm, determine
 - (i) Altitude of flight
 - (ii) Ground coordinates X and Y, if coordinates on the photograph are 34.5 mm and 24 mm respectively.
2. A photographic survey was carried out to a scale of 1:20000. Find the error in the height where given that there is an error of 0.15 mm in measuring the parallax of the point. Given, $f = 200$ mm, size of photograph is 250 mm \times 250 mm and overlap is 60%.
3. In a pair of overlapping vertical photographs, the base distance $B = 600$ m. If the altitude of the horizontal flight was 750 m above the mean sea level, what will be the height of chimney, if its base is 200 m above the mean sea level and the difference of parallax between its top and bottom is 12.8 mm? Given that the focal length of the camera was 180 mm.

1.8.3 Measurement of Parallax

- It is shown in the previous article that if the difference in parallax Δp is known, the difference in elevation Δh between the two points can be calculated.
- To measure the difference of parallax Δp between a pair of overlapping photographs, an instrument called as *Parallax Bar* is used.
- The measurement of parallax by parallax bar is based on the principle of *floating mark*.

➤ Fig beside shows the principle of floating point.

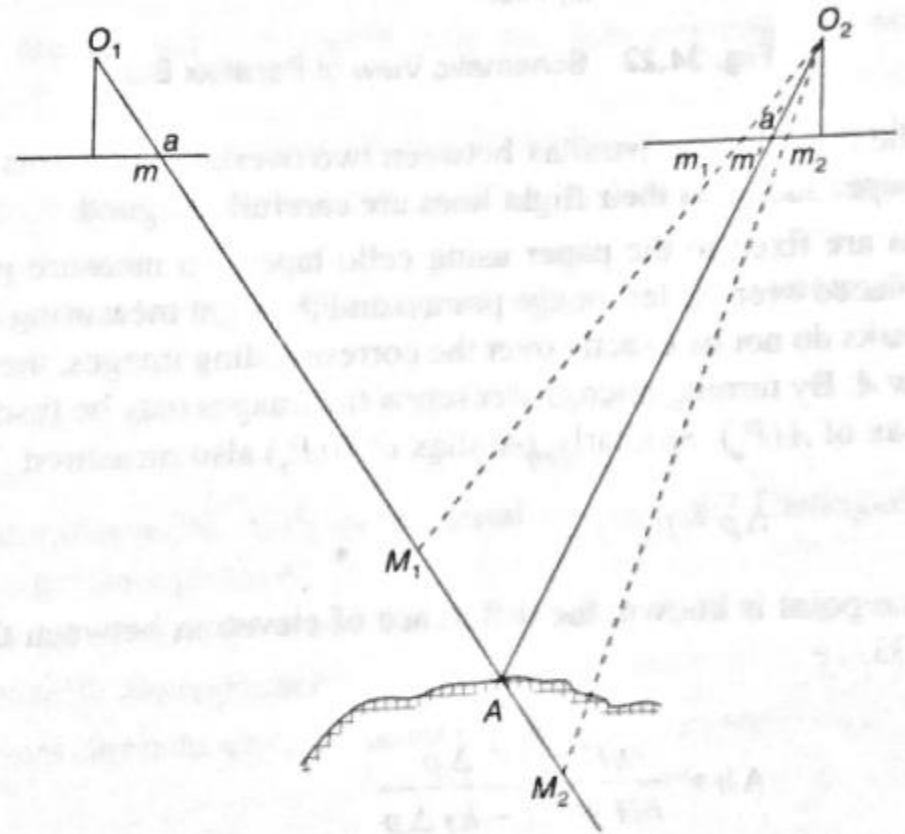
➤ On the left side photograph, the ground point A appears at m and on the right side photograph, it appears at m'.

➤ If the left side photograph is held at fixed position and right hand side photograph is slightly pushed to get m' at position m₁ the ground point A will shift to position M₁.

➤ Similarly, if m' is moved to m₂ the ground point will move to M₂.

➤ Thus if and only if the images of the two points are fused, the points are

on the ground. If the images are of different points they can not be fused to a point. This principle has been used in the parallax bar.



Principle of Floating Mark

- A schematic view of parallax bar is shown in the fig. below. It consists of a proper bar which holds a fixed plate of transparent material near the left end and a movable plate to the right end.
- The right plate can be moved by operating the micrometer screw which measures the movement to an accuracy of 0.01 mm.

- For measuring the difference in parallax between two overlapping points, the photographs are kept on a large sheet of paper such that their flight lines are carefully aligned.

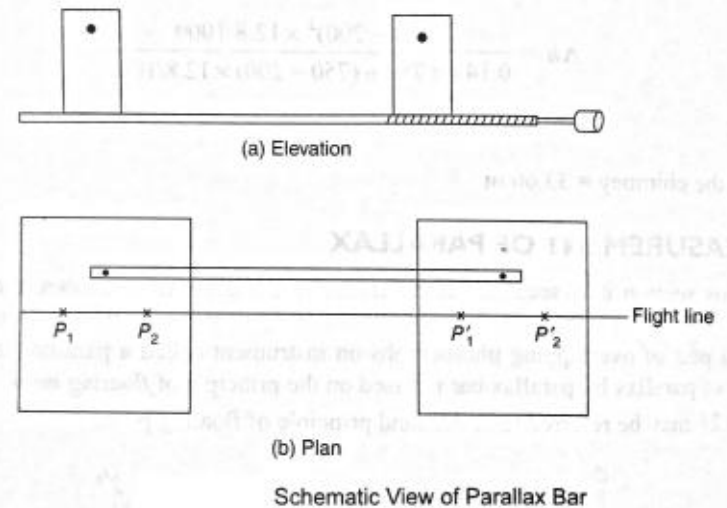
- The photographs are now fixed in its position using cello tapes.

- To measure parallax of A, the left measuring mark is placed over the left image point m and the right measuring mark over the right image m'.

- If these marks do not lie exactly over the corresponding images, the floating mark appear to lie above or below A.

- By turning micrometer screw, the images may be fused and the micrometer reading gives parallax of A (p_a) on one photograph. Similarly, parallax of A (p_b) on the other photograph can be measured. Then, $\Delta p = p_a - p_b$. The difference of elevation can be found using the equation. Value of b may vary for the two points, mean value is adopted.

$$\Delta h = \frac{(H - h)^2 \Delta p}{bH + (H - h)\Delta p}$$



1.9 Stereoscopic Plotting

- A stereo plotter, also known as multiplex, is an instrument used for preparing maps of high precision from aerial photographs.
- It is an optical and mechanical apparatus.
- Its function is to create a three dimensional scale optical model of overlapping area.
- It provides means to view the spatial model and provide a means for measuring heights and plotting orthotropic (map) view.
- It is used by big organizations.

1.10 Errors in Aerial Survey

- Failure to maintain altitude of aircraft.
- Inaccuracies in locating and transferring the principal points.
- Failure to see the floating mark correctly on the point.
- Inaccurate orientation of photographs while preparing maps.
- Neglecting the tilt of camera.
- Failure to provide proper control points.
- Reading the scales incorrectly.